

## TEACHING THROUGH PROBLEM-SOLVING: THREE-PART LESSON FORMAT

An information- and technology-based society requires individuals who are able to think critically about complex issues, people who can “analyze and think logically about new situations, devise unspecified solution procedures, and communicate their solution clearly and convincingly to others” (Baroody, 1998, p. 2-1). To prepare students to function in such a society, teachers have a responsibility to promote in their classrooms the experience of problem-solving processes and the acquisition of problem-solving strategies, and to foster in students positive dispositions towards problem solving.

Ontario Ministry of Education (nd). *Problem Solving and Communication in The Guide to Effective Instruction in Mathematics, K to 6*, p.3

The Ministry of Education recommends that Kindergarten students engage in explicit mathematics learning for a minimum of 20 minutes daily (40 minutes every other day) and Grade 1-8 students have a minimum of 60 minutes of math daily.

### BEFORE: GETTING STARTED / MINDS ON

#### Teacher Actions

- activate students’ prior knowledge
- mentally engage students in the problem-solving situation by posing a thought-provoking problem
- model tool and strategy selection as appropriate
- gather diagnostic and/or formative assessment data through observation and questioning
- discuss and clarify the task, including having students restate the problem in their own words and ask questions
- establish expectations
- establish a positive mathematics classroom climate

#### Student Actions

- participate in discussions
- propose strategies
- question the teacher and their classmates
- make connections to and reflect on prior learning

### DURING: WORKING ON IT / ACTION

#### Teacher Actions

- facilitate student learning by:
  - providing hints and suggestions
  - encouraging testing of ideas
  - suggesting extensions or generalizations
  - asking probing questions
  - answering students’ questions to clarify mathematical misconceptions (while avoiding providing a solution to the problem)
  - encouraging students to represent their thinking
- observe and assess
- reconvene the whole group if significant questions arise
- encourage students to clarify ideas and to pose questions to other students (math talk)
- make connections with literacy and learning for life

#### Student Actions

- participate actively in whole group, small group, or independent settings
- explore and develop strategies and concepts
- select appropriate tools and strategies
- represent their thinking in a variety of ways
- develop and reflect upon alternative solutions
- engage in metacognition
- communicate their understanding to their classmates and the teacher

### AFTER: REFLECT AND CONNECT / CONSOLIDATE AND DEBRIEF

#### Teacher Actions

- facilitate whole class discussion and sharing by:
  - bringing students back together to share and analyze solutions and clarify misunderstandings
  - encouraging students to explain a variety of solution strategies
  - asking students to defend their procedures and justify their answers
  - engaging all class members
  - summarizing the discussion and emphasizing key points or concepts, i.e., “pulling out the math”
- use praise cautiously
- connect strategies and solutions to similar types of problems in order to help students generalize concepts
- check for conceptual understanding

#### Student Actions

- justify and explain their thinking and understanding with clarity and precision
- compare a variety of concrete, pictorial, and numerical representations
- listen and contribute to reflections on alternative approaches and different solutions, as well as extensions and connections
- reflect on their learning

Assessment is an on-going awareness of students’ learning and their needs, rather than an occasional event in the program.

Ontario Ministry of Education (nd). *Foundations of Math Instruction in The Guide to Effective Instruction in Mathematics, K to 6*, p.3

## PROBLEM SOLVING MODEL

### Understand the Problem (the exploratory stage)

- reread and restate the problem
- identify the information given and the information that needs to be determined

**Communication:** talk about the problem to understand it better

### Make a Plan

- relate the problem to similar problems solved in the past
- consider possible strategies
- select a strategy or a combination of strategies

**Communication:** discuss ideas with others to clarify which strategy or strategies would work best

### Carry Out the Plan

- execute the chosen strategy
- do the necessary calculations
- monitor success
- revise or apply different strategies as necessary

**Communication:** talk about the problem to understand it better

- draw pictures or use manipulatives to represent interim results
- use words and symbols to represent the steps in carrying out the plan or doing the calculations
- share results of computer or calculator operations

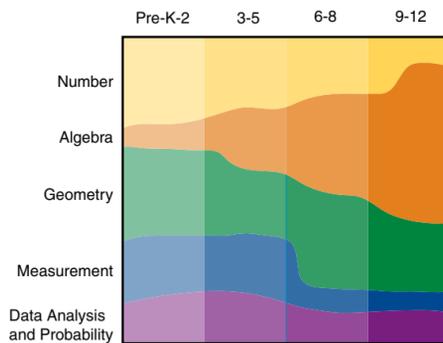
### Look Back at the Solution

- check the reasonableness of the answer
- review the method used: Did it make sense? Is there a better way to approach the problem?
- consider extensions or variations

**Communication:** describe how the solution was reached, using the most suitable format, and explain the solution

The Ontario Mathematics Curriculum, Grades 1 – 8 (Revised), p. 13

## RELATIVE EMPHASIS OF STRANDS



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## SUMMARY

Mathematical activity in a mathematics classroom is characterized by:

- computing
- recalling facts
- manipulating
- using manipulatives and technology
- exploring
- hypothesizing
- inferring/concluding

## LEARNING TOOLS (Manipulatives & Technology)

	Pre K-3	4-6	7-9	10-12
1 cm Interlocking cubes	✓	✓	✓	
2 cm Interlocking cubes		✓	✓	✓
Abacus	✓			
Angle Measurement Tools		✓		✓
Algebra Tiles		✓	✓	✓
Attribute Blocks	✓			
Balance and Weights	✓	✓	✓	
Base 10 Materials	✓	✓	✓	✓
Clocks	✓	✓	✓	
Coloured Tiles	✓	✓	✓	✓
Compasses	✓	✓	✓	✓
Cuisinaire Rods	✓	✓	✓	✓
Dice	✓	✓	✓	✓
Fraction Pieces/Rings/Circles	✓	✓	✓	✓
Geoboards	✓	✓	✓	✓
Hundreds Chart/Carpet/Board	✓	✓	✓	✓
Materials for Counting/Sorting	✓	✓	✓	✓
Linear Measurement Tools	✓	✓	✓	✓
Miras	✓	✓	✓	✓
Money	✓	✓	✓	✓
Pattern Blocks	✓	✓	✓	✓
Pentominoes	✓	✓	✓	✓
Polygons + Power Pack	✓	✓	✓	✓
Polydrons	✓	✓	✓	✓
Power Polygons	✓	✓	✓	✓
3 D Models	✓	✓	✓	✓
Tangrams	✓	✓	✓	✓
TI-10 Calculators	✓	✓	✓	✓
TI-15 Calculators		✓	✓	✓
TI Graphing Calculators		✓	✓	✓
Two Coloured Counters	✓	✓	✓	✓
Volume Measurement Tools	✓	✓	✓	✓

## USEFUL WEBSITES

**Ontario Ministry of Education:** [www.edu.gov.on.ca/eng](http://www.edu.gov.on.ca/eng)  
 - curriculum documents (K – 12), Leading Math Success (7 – 12) materials (GSP files, lessons and assessments, professional learning materials), and other current ministry materials

**eworkshop:** <http://www.eworkshop.on.ca/edu/core/cfm>

- all the Literacy and Numeracy Secretariat materials, including modules (with videos, printable documents, and scaffolded activities)

**Curriculum Services Canada:** [www.curriculum.org](http://www.curriculum.org)

- all the webcasts related to numeracy from LNS, as well as a wealth of other resources suitable for Ontario classrooms

**Ontario Association for Mathematics Education (OAME):**

[www.oame.on.ca](http://www.oame.on.ca)

- resources developed by Ontario teachers for Ontario classrooms, along with current information about professional learning opportunities in local chapters

**National Council of Teachers of Mathematics (NCTM):**

[www.nctm.org](http://www.nctm.org)

- research and current thinking around mathematics education, also a wealth of materials for classroom use

**Growing Accessible Interactive Networked Supports (GAINS):**

[www.edugains.ca](http://www.edugains.ca)

- Ontario based resources for classroom use and professional learning, also dialogue opportunities for teachers (7 – 12)

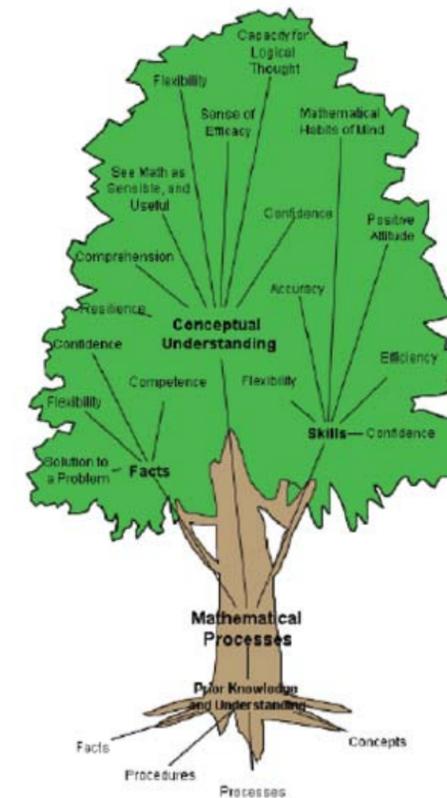
- revising/revisiting/reviewing/reflecting
- explaining
- making convincing arguments, explanations, and justifications
- using mathematical language, symbols, forms, and conventions
- integrating narrative and mathematical forms
- interpreting mathematical instructions, charts, drawings, graphs
- representing a situation mathematically
- selecting and sequencing procedures

Ontario Ministry of Education (2005). *Developing Mathematical Literacy*. p. 14.



# MATHEMATICAL PROFILE

## A Balanced Approach to Promote Mathematical Literacy K-12



“Mathematics literacy is an individual’s capacity to identify and understand the role that mathematics plays in the world, to make well-founded mathematical judgements and to engage in mathematics, in ways that meet the needs of that individual’s current and future life as a constructive, concerned and reflective citizen.” (Measuring Up, OECD PISA Study, 2001, p. 10)

*Leading Math Success; Developing Mathematical Literacy* p.12.

Based on materials from:

*The Ontario Ministry of Education | The Literacy and Numeracy Secretariat | National Council of Teachers of Mathematics (NCTM)*

*Elementary and Middle School Mathematics: Teaching Developmentally by John A. Van de Walle*

Revised 2009

## AN EFFECTIVE MATHEMATICS ENVIRONMENT:

- fosters positive mathematical attitudes
- focuses on conceptual understanding as well as procedural fluency
- focuses on important mathematical concepts or “big ideas”
- allows opportunities for students to take risks, explore, investigate, and communicate mathematically
- acknowledges and builds upon students’ prior knowledge
- provides a culture and climate for learning mathematics through problem solving
- develops and sustains metacognition
- provides opportunities for students to develop mathematical understanding through their strengths, learning styles, interests, and developmental levels

## EFFECTIVE MATHEMATICS INSTRUCTION:

- is focused on having students make sense of mathematics
- is based on problem solving and investigation of important mathematical concepts
- begins with the students’ understanding and knowledge of the topic
- includes students as active rather than passive participants in their learning
- has students communicate and investigate their thinking through ongoing discussion
- includes all students, whether in the choice of problems or in the communicating of mathematical ideas
- incorporates ongoing assessment of student understanding to guide future instruction

Ontario Ministry of Education (2004). *Teaching and Learning Mathematics: The Report of the Expert Panel on Mathematics in Grades 4 to 6 in Ontario*. p. 8.

Mathematical literacy is valued for many different reasons. Mathematics provides powerful numeric, spatial, temporal, symbolic, and communicative tools. Mathematics is needed for “everyday life” to assist with decision making. As John Allen Paulos pointed out in his book, *Innumeracy* (2001), “... numeracy is the ability to deal with fundamental notions of number and chance in order to make sense of mathematical information presented in everyday contexts.”

Mathematics and problem solving are needed in the workplace for many professions such as health science workers or graphic artists, as well as statisticians and engineers. Mathematics is also, ultimately, a cultural and intellectual achievement of humankind and should be understood in its aesthetic sense (NCTM, 2000). All people have a right of access to the domain of mathematics.

Ontario Ministry of Education (2005). *Developing Mathematical Literacy*. p. 12.

PROCESS	DESCRIPTOR	PRIMARY	JUNIOR	INTERMEDIATE	SENIOR
<p><b>Problem Solving</b></p> <p>Teacher provides numerous opportunities for students to solve problems and learn through problem solving, allowing students to connect mathematical ideas and to develop conceptual understanding. Problem solving forms the basis of effective mathematics programs and should be the mainstay of mathematical instruction.</p>	<ul style="list-style-type: none"> <li>• develop mathematical understanding and give meaning to skills and concepts in all strands</li> <li>• reason, communicate ideas, make connections, and apply knowledge and skills</li> <li>• collaborative sharing of ideas and strategies, promote accountable mathematical talk</li> <li>• use critical-thinking skills (estimating, evaluating, classifying, assuming, recognizing relationships, hypothesizing, offering opinions with reasons, and making judgements)</li> </ul>	<p><i>Students will...</i></p> <p><i>apply developing problem-solving strategies as they pose and solve problems and conduct investigations, to help deepen their mathematical understanding</i></p> <ul style="list-style-type: none"> <li>• Teachers question students to allow them to determine whether an assertion is true and to extend their thinking</li> <li>• Teachers emphasize the importance of counter examples (i.e., use an example to disprove a statement)</li> <li>• Students access a variety of tools in order to develop the ability to choose an appropriate strategy while solving problems</li> </ul>	<p><i>Students will...</i></p> <p><i>develop, select and apply problem-solving strategies, as they pose and solve problems and conduct investigations, to help deepen their mathematical understanding</i></p> <ul style="list-style-type: none"> <li>• Teachers establish the classroom as a mathematical community that is continually developing, testing, and applying conjectures about mathematical relationships</li> <li>• Students need opportunities to revise, expand, and update their generalizations</li> <li>• Students are exposed to a variety of problem solving situations in order to develop the ability to choose appropriate strategies and tools</li> </ul>	<p><i>Students will...</i></p> <p><i>develop, select, apply, and compare a variety of problem-solving strategies as they pose and solve problems and conduct investigations to help deepen their mathematical understanding</i></p> <ul style="list-style-type: none"> <li>• Teachers regularly engage students in thinking and reasoning in the classroom to investigate mathematical relationships</li> <li>• Students generate and organize data, validate or refute conjectures</li> <li>• Students are exposed to a variety of problem solving situations in order to develop the ability to choose appropriate strategies and tools</li> <li>• Students see the importance of inquiry in other subjects, such as social studies, language, science</li> </ul>	<p><i>Students will...</i></p> <p><i>develop, select, apply, compare, and adapt a variety of problem-solving strategies as they pose and solve problems and conduct investigations, to help deepen their mathematical understanding</i></p> <ul style="list-style-type: none"> <li>• Teachers provide students with the opportunity to learn important content through explorations of problems and to learn and practice a wide range of strategies using a variety of assumptions</li> <li>• Students seek, formulate, and critique explanations within a classroom climate of discussing, questioning, and listening</li> <li>• Students are exposed to a variety of problem solving situations in order to develop the ability to choose appropriate strategies and tools.</li> </ul>
<p><b>Reasoning and Proving</b></p> <p>Students explore phenomena, develop ideas, make mathematical conjectures, and justify results. Teachers draw on students’ natural ability to reason to help them learn to reason mathematically.</p>	<ul style="list-style-type: none"> <li>• reason from the evidence found in explorations and investigations or from what is already known to be true, and recognize the characteristics of an acceptable argument in the mathematics classroom</li> <li>• revisit conjectures that have been found to be true in one context to see if they are always true</li> </ul>	<p><i>apply developing reasoning skills (e.g., pattern recognition, classification) to make and investigate conjectures (e.g., through discussion with others)</i></p> <ul style="list-style-type: none"> <li>• Students conjecture (predict a result) and prove or disprove their conjecture</li> <li>• Teachers model use of logical language, including not, and, or, all, some, if ... then, because</li> </ul>	<p><i>develop and apply reasoning skills (e.g., classification, recognition of relationships, use of counterexamples) to make and investigate conjectures and construct and defend arguments</i></p> <ul style="list-style-type: none"> <li>• Teachers ensure that conjectures and mathematical arguments that have been developed are applied to further work</li> <li>• Students are responsible for articulating their own reasoning and for understanding the reasoning of others</li> </ul>	<p><i>develop and apply reasoning skills (e.g., recognition of relationships, generalization through inductive reasoning, use of counterexamples) to make mathematical conjectures, assess conjectures and justify conclusions, and plan and construct organized mathematical arguments</i></p> <ul style="list-style-type: none"> <li>• Students begin to develop facility with deductive reasoning</li> <li>• Teachers communicate the strengths and limitations of inductive reasoning, including then, because</li> </ul>	<p><i>develop and apply reasoning skills (see specific course curriculum for examples) to make mathematical conjectures, assess conjectures and justify conclusions, and plan and construct organized mathematical arguments</i></p> <ul style="list-style-type: none"> <li>• Teachers provide opportunities for students to refine their deductive and inductive reasoning skills</li> <li>• Students are assisted as they develop logical structure to their arguments</li> </ul>
<p><b>Reflecting</b></p> <p>Good problem solvers regularly and consciously reflect on and monitor their own thought processes. Reflecting on their own thinking and the thinking of others helps students make important connections and internalize a deeper understanding of the mathematical concepts involved.</p>	<ul style="list-style-type: none"> <li>• share strategies, defend procedures, justify answers, and clarify any misunderstandings</li> <li>• consider reasonableness of answers</li> <li>• self-monitor progress while problem solving and revise as necessary</li> <li>• apply and extend knowledge to new situations</li> </ul>	<p><i>demonstrate that they are reflecting on and monitoring their thinking to help clarify their understanding as they complete an investigation or solve a problem (by explaining to others why they think their solution is correct)</i></p> <ul style="list-style-type: none"> <li>• Teachers engage in the think-aloud process to model reflecting on and monitoring thinking</li> <li>• Teachers ask questions in order to engage students in examination of their own thought processes (metacognition)</li> <li>• Students begin to reflect on and monitor their thinking throughout mathematical activities</li> </ul>	<p><i>demonstrate that they are reflecting on and monitoring their thinking to help clarify their understanding as they complete an investigation or solve a problem (e.g., by comparing and adjusting strategies used, by explaining why they think their results are reasonable, by recording their thinking in a math journal)</i></p> <ul style="list-style-type: none"> <li>• Teachers engage in the think-aloud process to model reflecting on and monitoring thinking</li> <li>• Students become increasingly adept at asking questions in order to examine their own thought processes (metacognition)</li> <li>• Students regularly reflect on and monitor their thinking throughout mathematical activities</li> </ul>	<p><i>demonstrate that they are reflecting on and monitoring their thinking to help clarify their understanding as they complete an investigation or solve a problem (e.g., by assessing the effectiveness of strategies and processes used, by proposing alternative approaches, by judging the reasonableness of results, by verifying solutions)</i></p> <ul style="list-style-type: none"> <li>• Teachers engage in the think-aloud process to model reflecting on and monitoring thinking</li> <li>• Students become increasingly independent at asking questions in order to examine their own thought processes (metacognition)</li> <li>• Students regularly reflect on and monitor their thinking throughout mathematical activities</li> </ul>	<p><i>demonstrate that they are reflecting on and monitoring their thinking to help clarify their understanding as they complete an investigation or solve a problem (see specific course curriculum for examples)</i></p> <ul style="list-style-type: none"> <li>• Teachers engage in the think-aloud process to model reflecting on and monitoring thinking</li> <li>• Students independently ask questions in order to examine their own thought processes (metacognition)</li> <li>• Students regularly reflect on and monitor their thinking throughout mathematical activities</li> </ul>
<p><b>Selecting Tools and Computational Strategies</b></p> <p>Students develop the ability to select the appropriate tool (concrete, visual, or electronic) and computational strategies to perform particular mathematical tasks, to investigate mathematical ideas, and to solve problems.</p>	<ul style="list-style-type: none"> <li>• new technology/manipulatives are introduced and modeled, allowing exploration and confidence building</li> <li>• a range of tools are available for use during instruction and assessment</li> <li>• tools are used to develop and communicate understanding of new concepts, as well as to strengthen existing knowledge</li> <li>• select appropriate computational strategies within problem solving activities</li> <li>• perform mental computations and estimations</li> </ul>	<p><i>select and use a variety of concrete, visual, and electronic learning tools and appropriate computational strategies to investigate mathematical ideas and to solve problems</i></p> <ul style="list-style-type: none"> <li>• Students have access to a selection of tools for instruction, learning, and assessment opportunities</li> <li>• Students use calculators in order to apply mathematics beyond pencil and paper competency, rather than as a replacement for an understanding of estimation and a knowledge of number facts</li> <li>• Teachers model appropriate and effective use of tools</li> <li>• Teachers ensure that estimation is incorporated into everyday lessons (i.e., students make meaningful estimates in contextual activities)</li> </ul>	<p><i>select and use a variety of concrete, visual, and electronic learning tools and appropriate computational strategies to investigate mathematical ideas and to solve problems</i></p> <ul style="list-style-type: none"> <li>• Students have access to a selection of tools for instruction, learning, and assessment opportunities</li> <li>• Students use calculators in order to apply mathematics beyond pencil and paper competency, rather than as a replacement for an understanding of estimation and a knowledge of number facts</li> <li>• Teachers value and teach the effective and efficient use of tools</li> <li>• Teachers ensure that estimation is incorporated into everyday lessons (i.e., students make meaningful estimates in contextual activities)</li> </ul>	<p><i>select and use a variety of concrete, visual, and electronic learning tools and appropriate computational strategies to investigate mathematical ideas and to solve problems</i></p> <ul style="list-style-type: none"> <li>• Students have access to tools for instruction, learning, and assessment opportunities</li> <li>• Students use calculators in order to apply mathematics beyond pencil and paper competency, rather than as a replacement for an understanding of estimation and a knowledge of number facts and concepts</li> <li>• Teachers value and teach the effective and efficient use of tools</li> </ul>	<p><i>select and use a variety of concrete, visual, and electronic learning tools and appropriate computational strategies to investigate mathematical ideas and to solve problems</i></p> <ul style="list-style-type: none"> <li>• Students have access to tools for instruction, learning, and assessment opportunities</li> <li>• Teachers value and teach the effective and efficient use of tools</li> <li>• Students make conjectures and explore them using technology</li> <li>• Teachers and students regularly use technology to complete mechanical activities quickly in order to place more emphasis of on investigation and analysis of mathematical concepts</li> </ul>
<p><b>Connecting</b></p> <p>Experiences that allow students to make connections help them to grasp general mathematical principles. As they make connections, students begin to see that mathematics is more than a series of isolated skills and concepts and that learning from one area of mathematics can be applied to understand another.</p>	<ul style="list-style-type: none"> <li>• make connections between new and prior knowledge to make sense of new learning</li> <li>• make explicit connections between mathematical concepts and skills and those in other disciplines</li> <li>• make connections between different representations</li> </ul>	<p><i>make connections among simple mathematical concepts and procedures, and relate mathematical ideas to situations drawn from other contexts</i></p> <ul style="list-style-type: none"> <li>• Teachers ask questions that emphasize the mathematical aspects of everyday situations</li> <li>• Teachers present tasks in new contexts that revisit previously learned concepts and procedures</li> <li>• Students have numerous opportunities to apply their knowledge in various contexts</li> </ul>	<p><i>make connections among mathematical concepts and procedures, and relate mathematical ideas to situations or phenomena drawn from other contexts (e.g., other curriculum areas, daily life, sports)</i></p> <ul style="list-style-type: none"> <li>• Tasks allow students to explore and develop increasingly sophisticated mathematical connections</li> <li>• Teachers ensure that links between prior knowledge and new knowledge are explicitly made (i.e., multiplication as repeated addition)</li> <li>• Teachers plan and highlight opportunities for the application of mathematical skills in other subjects</li> </ul>	<p><i>make connections among mathematical concepts and procedures, and relate mathematical ideas to situations or phenomena drawn from other contexts (e.g., other curriculum areas, daily life, current events, art and culture, sports)</i></p> <ul style="list-style-type: none"> <li>• Teachers select questions that connect mathematical ideas within topics and across the curriculum and that allow students to build on their current knowledge</li> <li>• Students are provided with time, opportunity and encouragement to identify and use connections</li> <li>• Students revisit mathematical problems in order to connect new skills to prior knowledge and develop an appreciation for more complex models</li> </ul>	<p><i>make connections among mathematical concepts and procedures, and relate mathematical ideas to situations or phenomena drawn from other contexts (see specific course curriculum for examples)</i></p> <ul style="list-style-type: none"> <li>• Teachers select questions that connect mathematical ideas within topics and across the curriculum and that allow students to build on their current knowledge</li> <li>• Students are encouraged to reflect upon and compare solutions to identify connections</li> <li>• Students revisit mathematical problems in order to connect new skills to prior knowledge and develop an appreciation for more complex models</li> </ul>
<p><b>Representing</b></p> <p>Learning the various forms of representation helps students to understand mathematical concepts and relationships. It also helps them to communicate their thinking, arguments, and understandings, recognize connections among related mathematical concepts, and use mathematics to model and interpret realistic problem situations.</p>	<ul style="list-style-type: none"> <li>• represent mathematical ideas and relationships and model situations using concrete materials, pictures, diagrams, graphs, tables, numbers, words, and symbols.</li> <li>• create, share, and interpret multiple representations</li> <li>• when concepts are represented in various ways, flexibility in thinking about those concepts is developed</li> </ul>	<p><i>create basic representations of simple mathematical ideas (e.g., using concrete materials; physical actions, such as hopping or clapping; pictures; numbers; diagrams; invented symbols), make connections among them, and apply them to solve problems</i></p> <ul style="list-style-type: none"> <li>• Teachers create an environment that encourages, supports, and accepts multiple representations of a single problem</li> <li>• Students develop and use multiple representations effectively</li> <li>• Teachers develop understanding by connecting standard notations to the methods and thinking of the students</li> <li>• Students recognize, through discussion, that any representation is subject to multiple interpretations</li> </ul>	<p><i>create a variety of representations of mathematical ideas (e.g., by using physical models, pictures, numbers, variables, diagrams, graphs, onscreen dynamic representations), make connections among them, and apply them to solve problems</i></p> <ul style="list-style-type: none"> <li>• Teachers emphasize the use of multiple models and highlight the effectiveness of specific models in specific situations</li> <li>• Students work with representations in various contexts and move between representations to understand how representations model mathematical ideas and relationships</li> <li>• Teachers develop understanding by connecting standard notations to the methods and thinking of the students</li> <li>• Students recognize that some representations are more efficient in specific contexts</li> </ul>	<p><i>create a variety of representations of mathematical ideas (e.g., numeric, geometric, algebraic, graphical, pictorial, onscreen dynamic representations), connect and compare representations, and select and apply the appropriate representations to solve problems</i></p> <ul style="list-style-type: none"> <li>• Teachers emphasize the use of multiple models and highlight the effectiveness of specific models in specific situations</li> <li>• Students move from individual (non-standard) models to conventional models with the timely and precise support of the teacher</li> <li>• Students have opportunities to develop confidence in creating and selecting models</li> </ul>	<p><i>create a variety of representations of mathematical ideas (see specific course curriculum for examples), connect and compare representations, and select and apply the appropriate representations to solve problems</i></p> <ul style="list-style-type: none"> <li>• Teachers highlight what different representations convey and emphasize the importance of selecting appropriate models within a given context and for a specific purpose</li> <li>• Students have access to a variety of tools in order to develop the ability to choose an appropriate strategy</li> <li>• Students have opportunities to develop confidence in creating and selecting models</li> <li>• Students develop their mathematical conceptual understanding by examining the information conveyed by different representations</li> </ul>
<p><b>Communicating</b></p> <p>Communication is the process of expressing mathematical ideas and understanding orally, visually, and in writing, using numbers, symbols, graphs, diagrams, pictures, and words.</p>	<ul style="list-style-type: none"> <li>• communicate for various purposes and for different audiences</li> <li>• reflect upon and clarify ideas, understanding of mathematical relationships, and mathematical arguments through oral, visual, and written communication</li> <li>• proper use of symbols, vocabulary, and notations is modeled and used in oral, visual, and written form</li> </ul>	<p><i>communicate mathematical thinking orally, visually, and in writing, using everyday language, a developing mathematical vocabulary, and a variety of representations</i></p> <ul style="list-style-type: none"> <li>• Teachers model appropriate conventional vocabulary</li> <li>• Students develop their mathematical literacy by talking with, and listening to, their peers during the problem solving process</li> <li>• Students use invented symbols and notations prior to the teacher explicitly connecting these invented symbols and notations to standard notation</li> </ul>	<p><i>communicate mathematical thinking orally, visually, and in writing, using everyday language, a basic mathematical vocabulary, and a variety of representations and observing basic mathematical conventions</i></p> <ul style="list-style-type: none"> <li>• Teachers model and discuss effective communication strategies, including appropriate use of mathematical vocabulary</li> <li>• Students engage in rich problem solving tasks that focus on the key learnings, providing them with situations that are worthy of their conversation and thought</li> <li>• Teachers plan well posed questions in order to elicit, extend, and challenge students’ thinking, thus allowing insight into students’ understanding</li> </ul>	<p><i>communicate mathematical thinking orally, visually, and in writing, using mathematical vocabulary and a variety of appropriate representations, and observing mathematical conventions</i></p> <ul style="list-style-type: none"> <li>• Teachers develop a culture in which explaining, questioning, debating, and sense making are natural and expected activities</li> <li>• Students share ideas and seek clarification until they understand</li> <li>• Teachers monitor communication to provide guidance through questioning, to encourage use of appropriate language, and to ensure equal opportunities to participate for all students</li> </ul>	<p><i>communicate mathematical thinking orally, visually, and in writing, using precise mathematical vocabulary and a variety of appropriate representations, and observing mathematical conventions</i></p> <ul style="list-style-type: none"> <li>• Teachers encourage classroom conversations at an appropriate level of discourse and mathematical argumentation</li> <li>• Students become more precise in their written work</li> <li>• Students are expected to read, discuss, and write increasingly technical text</li> <li>• Communication is assessed on an on-going basis for clarity of thought as well as use of mathematical conventions</li> </ul>